Renormalization group studies of the thermodynamics and kinetics of Ising models

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Motivation

- The thermodynamic Ising model has been successfully studied with the Renormalization group ($\mathcal{RG}$) [1–4].
- Numerical methods for updating coupling constants and describing the thermodynamic model using rescaled representations of itself are presented by Swendsen [3, 4].
- Our aim is to investigate the rescaling behaviour of the rates assigned to events in the kinetic Ising model evolved under Glauber dynamics [5, 6].
- The dynamical scaling behaviour of the Ising model is considered hard to calculate accurately.
- A method for evaluating the rescaled kinetic rates would enable simulation on greater length and time scales.

Thermodynamic Ising Model

- The two dimensional thermodynamic Ising model is a lattice occupied by $|\uparrow\rangle$ and $|\downarrow\rangle$ spins denoted by the set $\{\sigma_i = \pm 1\}$ [2, 7].
- The system undergoes an order-disorder phase transition at a critical temperature T_c and is scale invariant at this point.
- An initial microstate is placed in contact with a thermal bath at temperature T and evolved through Monte Carlo simulation methods [8–12].
- Thermodynamic quantities are sampled until their mean converges.

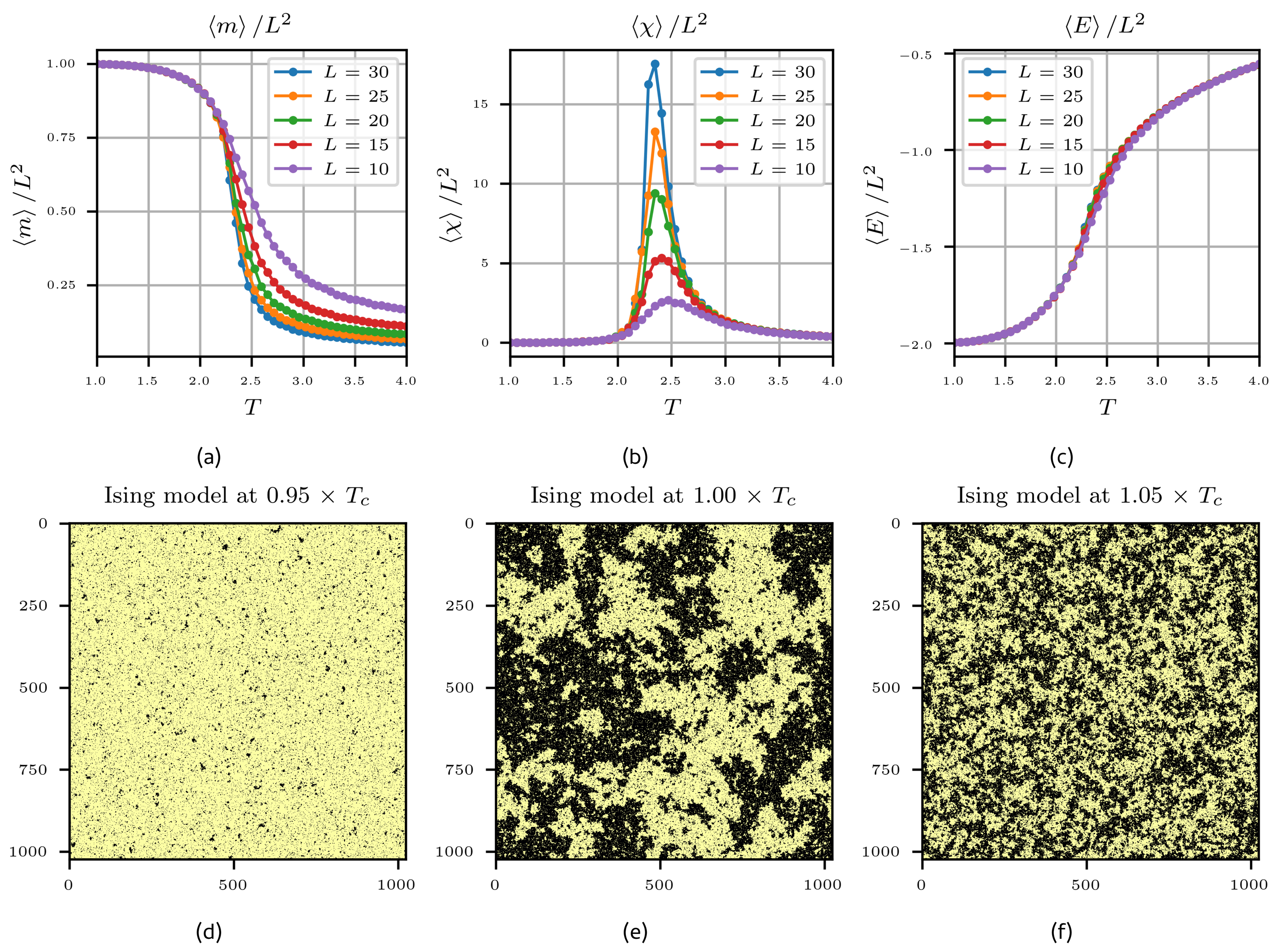

Figure 1: **(a)–(c)** Show plots of the expectation values for magnetization, magnetic susceptibility and energy per site on $L \times L$ lattices. Each data point is averaged over 2×10^4 Monte Carlo steps per site using the Metropolis method described in [8]. The peak in the susceptibility plot occurs at the second order phase transition. **(d)–(f)** Show snapshots of microststates (taken after 2×10^3 Monte Carlo steps per site) representing the Ising model in equilibrium with baths of temperatures **(d)** $0.95T_c$, **(e)** T_c and **(f)** $1.05T_c$.

Kinetic Ising Model

- The zero temperature Glauber dynamics of the Ising model involve quenching the model to zero temperature from above criticality [5, 6].
- In order to evolve the model, spin flip events are categorised by their associated energy change.
- Allowed events are flips that decrease or do not change the energy of the system.
- Events are then selected proportional to the rate at which they occur as a fraction of all potential events. This can be sampled by Kinetic Monte Carlo [13].
- As the model quenches from $T = \infty$ to $T = 0$ it exhibits a coarsening mosaic before reaching a stable configuration [6].
- A stable configuration is one with no possible events, i.e. a homogeneous system or spanning stripes of at least 2 sites in width [6].

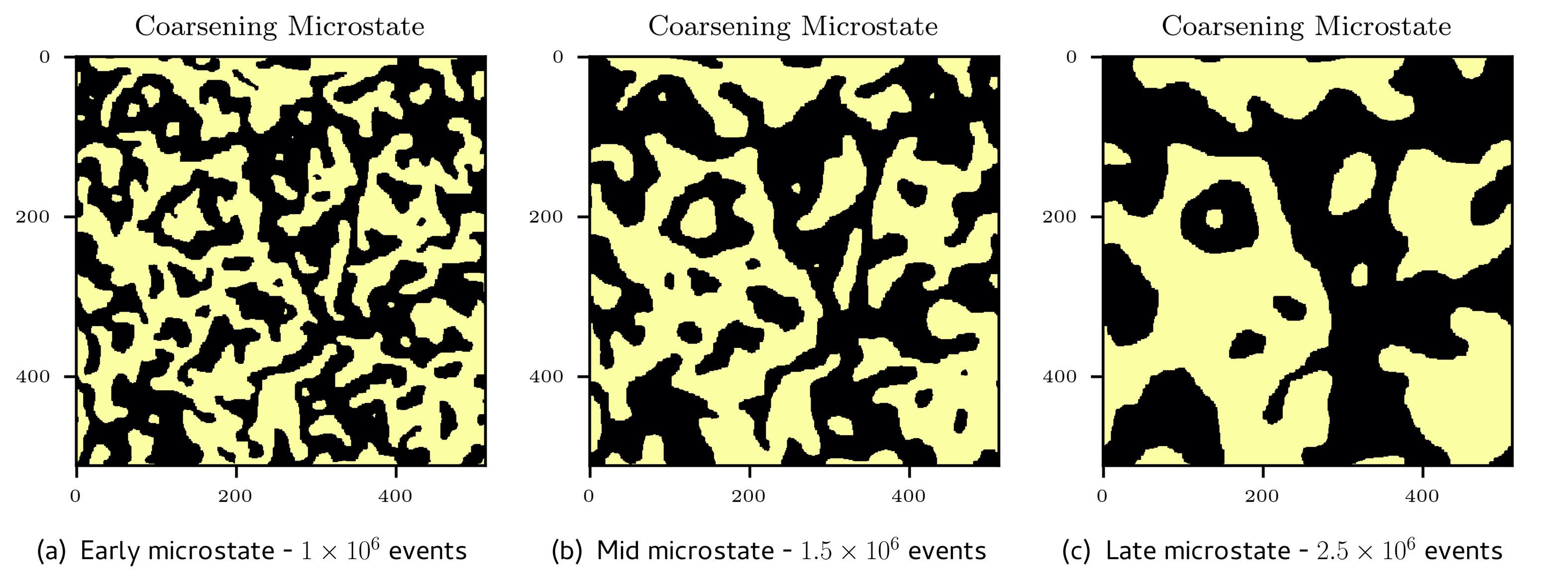

Figure 2: Shows snapshots of the zero temperature coarsening process after **(a)** 1×10^6, **(b)** 1.5×10^6 & **(c)** 2.5×10^6 events. The simulations are initialized with a random initial microstate corresponding to a model at $T = \infty$. The system finally evolves to homogeneity or stable stripes.

Renormalization Group – Thermodynamic

- Block spin transformations can be used in order to renormalize spin models such as the Ising model [1, 2].
- The real space $\mathcal{RG}$ transformations can be performed by dividing the model into blocks containing $b \times b$ spins and replacing these with a single spin matching the block majority.

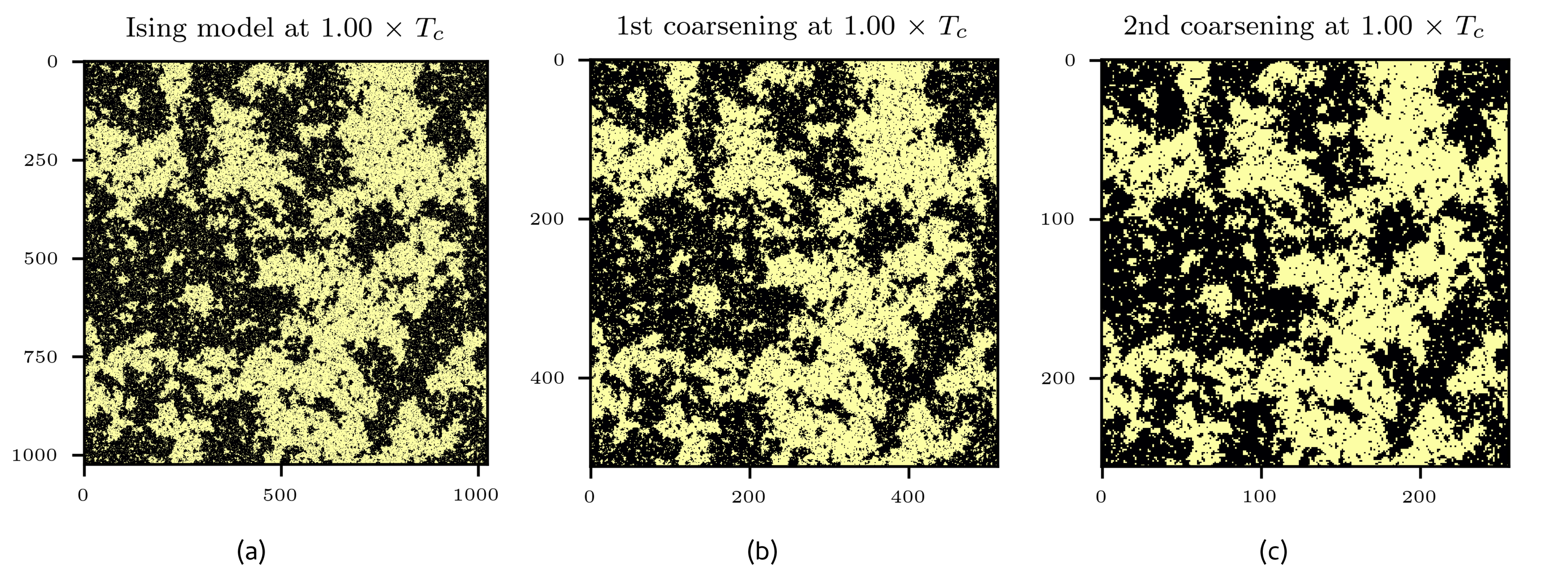

Figure 3: Shows the two dimensional Ising model at T_c after **(a)** 0, **(b)** 1 & **(c)** 2 $\mathcal{RG}$ transformations with a block size of $b = 2$. These transformations show the system to be scale invariant at criticality.

- The eigenvalues of the renormalization transformation can be evaluated by Monte Carlo sampling [3, 4].

Renormalization Group – Kinetic

- The scaling behaviour of the rates upon $\mathcal{RG}$ transformations in the kinetic Ising model is currently unknown.
- Obtaining rescaled rates would allow larger scale simulations to be represented in the same parameter space.
- Rescaled rates can be obtained by evolving the original model and monitoring changes on the rescaled model.
- Observing events on the rescaled model allows them to be categorised and time averaged in order to obtain their rate.
- The same methods for evolving the original model could then be used with these renormalized rates.

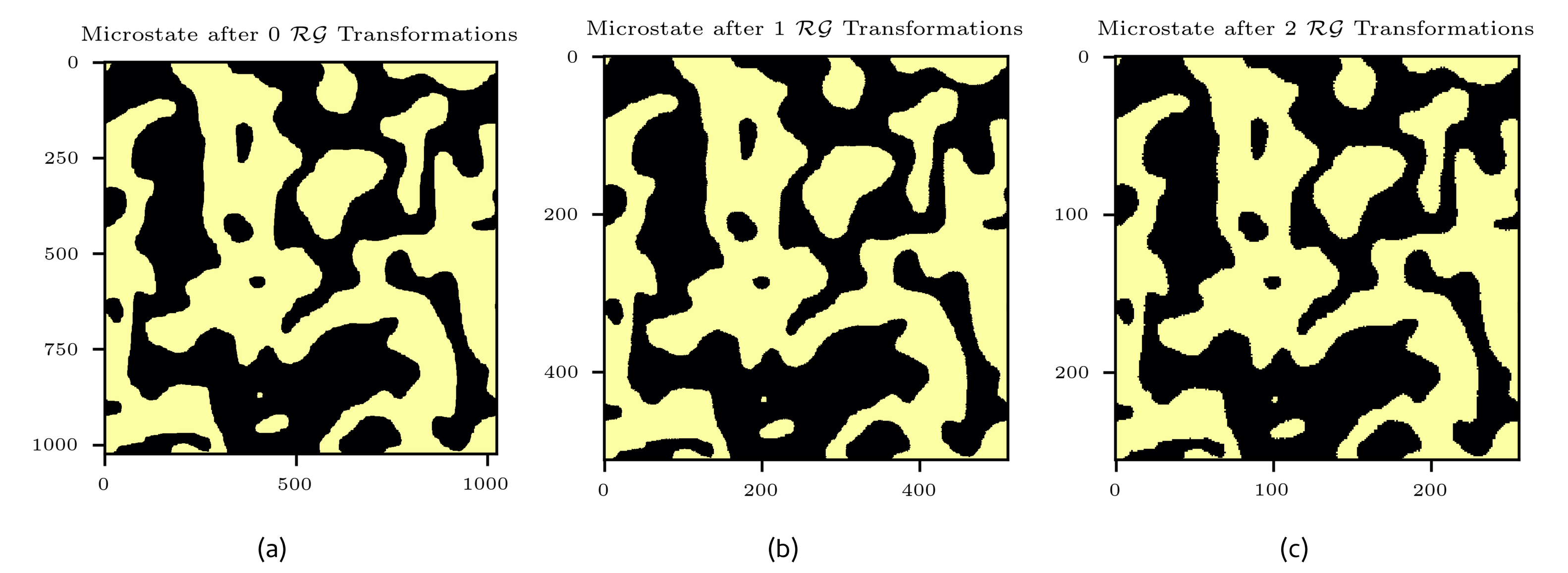

Figure 4: Shows snapshots of the zero temperature coarsening process at 1.5×10^7 events after **(a)** 0, **(b)** 1 & **(c)** 2 $\mathcal{RG}$ transformations.

References & Acknowledgements

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